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A new traffic model on compulsive lane-changing caused by off-ramp*

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In the field of traffic flow studies, compulsive lane-changing refers to lane-changing (LC) behaviors due to traffic rules or bad road conditions, while free LC happens when drivers change lanes to drive on a faster or less crowded lane. LC studies based on differential equation models accurately reveal LC influence on traffic environment. This paper presents a second-order partial differential equation (PDE) model that simulates both compulsive LC behavior and free LC behavior, with lane-changing source terms in the continuity equation and a lane-changing viscosity term in the momentum equation. A specific form of this model focusing on a typical compulsive LC behavior, the 'off-ramp problem', is derived. Numerical simulations are given in several cases, which are consistent with real traffic phenomenon.

Keywords: traffic flow model, compulsive lane-changing, off-ramp, fluid dynamics, numerical simulation

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1. Introduction

As an important driving behavior, lane-changing behavior (LC) has gained increasing attention in the past few years, and has become a remarkable problem in traffic flow studies. Recent papers revealed that LC is crucial for traffic relaxation[1,2] and safety.[3–7] Therefore, researches on LC problems may carry great significance.

Toledo,[8] Moridpour *et al.*,[9] and Zheng[10] have reviewed LC, which categorized LC behavior as free LC (i.e., discretionary LC) and compulsive LC (i.e., mandatory LC), according to how the decision of LC is made. Free LC is executed to improve driving conditions, such as changing to a faster lane to achieve higher speed, and changing to a less crowded lane to be more comfortable. Compulsive LC is executed when the driver has to leave the current lane due to certain traffic rules or bad road conditions, for example, the off-ramp problem: a driver who intends to leave the main road from the off-ramp ahead always moves into the right-hand lane or the auxiliary lane in advance to prepare for leaving (in right-driving traffics). This problem is modeled and studied in this paper.

In traffic flow studies, the continuum model is an effective tool and yields good results of studies about how driving behaviors affect the surroundings. Lighthill and Whitham,[11] and Richards[12] respectively originated the model by applying the continuity equation in fluid mechanics to traffic flow, known as the LWR theory. Payne built a second-order continuum model,[13] which has a similar form to the Navier-Stokes equation. Let ρ be the traffic density, u be the speed, then

$$\begin{aligned}&\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+\rho\frac{\partial u}{\partial x}=\Phi(x,t),\\&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{a^2}{\rho}\frac{\partial \rho}{\partial x}=\frac{1}{Tr}(U_e-u),\end{aligned}\qquad(1)$$

where $\Phi(x,t)$ is the traffic source term, a is the sonic speed, Tr is the time delay parameter, and U_{e} is the equilibrium speed function. Later, various models were built based on Payne's model.[14–17] Papageogiou's model adds a term to the momentum equation to simulate the influence of the ramp on the main road traffic:[14]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{a^2}{\rho}\frac{\partial \rho}{\partial x}=\frac{1}{Tr}(U_e-u)-\delta\frac{u\Phi}{\rho},\qquad(2)$$

where Φ is the on-ramp traffic flow per unit width of the ramp, and $\delta\in[0,1]$, is a dimensionless coefficient related to the speed difference between the ramp and the main road.

In the past few years, several continuum traffic models on free LC have been published. Laval and Daganzo adopted the speed difference between adjacent lanes to construct,[7] while Zhu and Wu[18] used the density difference instead. Ko *et al.*[19] integrated the two forms of the source term to fully characterize the free LC phenomenon. For a section with n lanes, numbered by $l=1,2,\ldots,n$ from left to right, the continuum equation and momentum equation of Ko et al.'s model are

$$\frac{\partial \rho_l}{\partial t}+u_l\frac{\partial \rho_l}{\partial x}+\rho_l\frac{\partial u_l}{\partial x}=\sum_{l'=l\pm1}\Phi_{\mathrm{free},l'l},\qquad(3)$$

$$\frac{\partial u_l}{\partial t}+u_l\frac{\partial u_l}{\partial x}+\frac{a^2}{\rho_l}\frac{\partial \rho_l}{\partial x}=\frac{1}{Tr}(U_{\mathrm{e}}-u)_l+f_{\mathrm{viscous},l},\qquad(4)$$

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(Note that terms where l' exceeds the range of l should not be included in the summation $\sum_{l'=l\pm1} \Phi_{\text{free},l'l}$, and likewise hereinafter), where $\Phi_{\text{free},l'l}$ is the free LC source term, defined as the number of vehicles that change from the l'-th lane to the l-th lane due to free LC per hour per kilometer, and is determined as

$$\begin{aligned}\Phi_{\text{free},l'l} = {} & C_1\left[\rho_{l'}u_{l'}\max(u_l-u_{l'},0)+\rho_l u_l\min(u_l-u_{l'},0)\right]\\ & +C_2\left[\rho_{l'}\max(\rho_{l'}-\rho_l,0)+\rho_l\min(\rho_{l'}-\rho_l,0)\right]. \end{aligned} \tag{5}$$

In the momentum equation (4), $f_{\text{viscous},l}$ is the viscosity term, derived through the analogy between free LC behavior and the viscosity of fluids. Its expression is

$$f_{\text{viscous},l} = \frac{1}{\rho_l}\sum_{l'=l\pm1}\Phi_{\text{free},l'l}\begin{cases}(u_{\text{f}}-u_l), & \rho_l \le 0.2\rho_{\text{jam}},\\ \left(-\frac{1}{4}u_{\text{f}}-u_l\right), & \rho_l > 0.2\rho_{\text{jam}}.\end{cases} \tag{6}$$

However, the LC models taking into account both compulsive LC and free LC behaviors rarely appear in the existing literature. This paper presents such a second-order differential equation model, derived based on Ko *et al.*'s free LC model. The remainder of this paper is organized as follows. In Section 2, a new continuum traffic model addressing compulsive LC and free LC together is derived. In Section 3, a numerical discretization method of the model is given. Numerical results on low-/high-density traffic and non-equilibrium traffic are presented in Section 4. Section 5 summarizes the results in the whole paper, and presents a prospect of further work.

2. Model

2.1. Derivation

Compulsive LC behaviors affect the density of the surrounding traffic environment, which can be depicted by adding a source term $\Phi_{\text{cmpl},l}$, the compulsive LC rate, to the right-hand side of the free LC continuity equation (3). $\Phi_{\text{cmpl},l}$ is defined as the number of vehicles that leave the l-th lane due to compulsive LC per hour per kilometer. A typical example of $\Phi_{\text{cmpl},l}$ is given by Eq. (8) below.

The vehicles that conduct compulsive LC may have different speeds from the target lane's average speed, which will change the speed profile nearby, and the free LC momentum equation (4) should be changed accordingly. Using Papageogiou's idea in his single-lane traffic model with ramp[14,15] for reference, we consider the influence of compulsive LC on momentum to be proportional to local speed and inversely proportional to local density, so we add a term $(u_l/\rho_l)/\Phi_{\text{cmpl},l}$ to the right-hand side of the momentum equation.

To summarize, for an n-lane road section, the partial differential equation (PDE) traffic model with considering both free and compulsive LC is as follows:

$$\begin{cases}\dfrac{\partial\rho_l}{\partial t}+u_l\dfrac{\partial\rho_l}{\partial x}+\rho_l\dfrac{\partial u_l}{\partial x}\\ =\sum_{l'=l\pm1}\Phi_{\text{free},l'l}-\Phi_{\text{cmpl},l},\\ \dfrac{\partial u_l}{\partial t}+u_l\dfrac{\partial u_l}{\partial x}+\dfrac{a^2}{\rho_l}\dfrac{\partial\rho_l}{\partial x}\\ =\dfrac{1}{Tr}(U_{\text{e}}-u)_l+f_{\text{viscous},l}+\dfrac{u_l}{\rho_l}\Phi_{\text{cmpl},l},\\ \Phi_{\text{free},l'l}=C_1\left[\rho_{l'}u_{l'}\max(u_l-u_{l'},0)+\rho_l u_l\min(u_l-u_{l'},0)\right]\\ \qquad +C_2\left[\rho_{l'}\max(\rho_{l'}-\rho_l,0)+\rho_l\min(\rho_{l'}-\rho_l,0)\right],\\ f_{\text{viscous},l}=\dfrac{1}{\rho_l}\sum_{l'=l\pm1}\Phi_{\text{free},l'l}\\ \qquad\cdot\begin{cases}(u_{\text{f}}-u_l), & \rho_l\le 0.2\rho_{\text{jam}},\\ \left(-\frac{1}{4}u_{\text{f}}-u_l\right), & \rho_l>0.2\rho_{\text{jam}}.\end{cases}\end{cases} \tag{7}$$

where ρ_l and u_l are the traffic density and the speed on lane l respectively; a is the sonic speed and Tr is the time delay parameter from Payne's model; $U_{\text{e}}(\rho)$ is the equilibrium speed function; $\Phi_{\text{free},l'l}$ and $\Phi_{\text{cmp},l}$ are the free LC rate and compulsive LC rate, which are both in units of veh/(h·km); $f_{\text{viscous},l}$ is the LC viscosity from Ko *et al.*'s model[19] in units of (km/h^2; C_1 is a constant parameter that addresses the effect of speed difference between adjacent lanes on the free LC rate, while C_2 is a constant parameter that addresses the influence of density difference on the free LC rate, C_1 and C_2 are in units of h/km^2 and (km/h)/veh respectively; u_{f} is the free flow speed; ρ_{jam} is the jam density.

In the off-ramp problem, attraction of the off-ramp ahead is the only considered compulsive LC factor. Thus the compulsive LC source term (namely, the compulsive LC rate) $\Phi_{\text{cmpl},l}$ in Eq. (7) is given as

$$\Phi_{\text{cmpl},l}=\Phi_{\text{ramp},l}-\Phi_{\text{ramp},l-1}, \tag{8}$$

where $\Phi_{\text{ramp},l}$ represents the number of vehicles that change from lane l to the adjacent right-hand lane per unit time and length due to the off-ramp ahead.

From the inspection of real traffic phenomenon, we find that the compulsive LC intensity has a general pattern: from the inlet to the upper stream of an off-ramp, the compulsive LC rate keeps increasing up to a peak at some point before the off-ramp, after which it decreases to zero at the point of the off-ramp. This rate then remains zero, because no compulsive LC behavior happens downstream of the off-ramp. According to this inspection, the expression of $\Phi_{\text{ramp},l}$ can be given as

$$\begin{aligned}\Phi_{\text{ramp},l} = {} & \alpha_l A\begin{cases}\operatorname{sech}(\gamma(x-x_0)), & x\le x_0,\\ \operatorname{sech}(\beta(x-x_0)), & x>x_0,\end{cases}\\ & (\gamma\ll\beta,\alpha_1<\alpha_2<\cdots<\alpha_n),\end{aligned} \tag{9}$$

where α_l is a dimensionless parameter, representing the relative intensity of occurrence for compulsive LC on lane l, and the inequalities in brackets indicate that the compulsive LC behavior caused by the off-ramp happens more frequently at the lane on the right-hand side than on the left-hand side. This inequality is analytical for the phenomenon observed in real traffic,[20] where the LC within two adjacent lanes increases from the median lane to the shoulder lane at the weaving section. The A is the overall intensity of occurrence, in the same units as Φ, i.e., veh/(h $\cdot$ km). The range of A value is determined through the experiment in Subsection 2.3. Parameters β and γ, in units of km^{-1}, are determined so that the curve shape of function $\Phi_{\mathrm{ramp},l}$ correctly reflects the pattern of off-ramp compulsive LC. Parameter x_0 in units of km is the point where the LC rate reaches its peak. A test drawing (Fig. 1 with $\gamma = 1.5$, $\beta = 150$, and $x_0^* = 0.6$) demonstrates that the function curve of $\Phi_{\mathrm{ramp},l}$ is in accordance with what we desire.

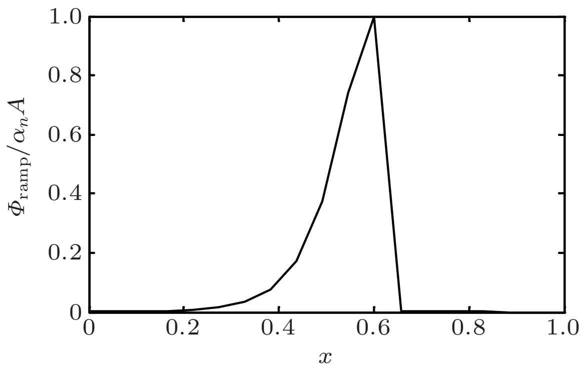

Fig. 1. Test drawing of Φ_{ramp} against x.

For the most right-hand side lane (i.e., the one that is adjacent to the auxiliary lane), the value of α_n can be determined through the following derivation. The definition of $\Phi_{\mathrm{ramp},n}$ implies that the integral of $\Phi_{\mathrm{ramp},n}$ with respect to x will be the total flow rate at which vehicles change to the auxiliary lane, and since all vehicles intending to leave the main lane must change to the auxiliary lane at some point before the ramp, the integral is thus the overall flow rate of leaving through the off-ramp. On the other hand, we define the overall intensity of occurrence A as the average flow rate at the off-ramp divided by the length of the road section. Therefore, the integral of $\Phi_{\mathrm{ramp},n}/A$ with respect to x should be 1. We can thus deduce that α_n can be obtained by the following expression:

$$\alpha_n = 1 \Big/ \left\{ \int_0^{x_0} \mathrm{sech}(\gamma(x - x_0))\,\mathrm{d}x + \int_{x_0}^{1} \mathrm{sech}(\beta(x - x_0))\,\mathrm{d}x \right\}, \tag{10}$$

where the integral terms can be calculated analytically by using the following formula:

$$\int_0^x \mathrm{sech}(\alpha\xi)\,\mathrm{d}\xi = \frac{2}{\alpha}\arctan\left(\tanh\left(\frac{\alpha}{2}x\right)\right). \tag{11}$$

2.2. Non-dimensionalization

Let u_{f} be the characteristic speed, ρ_{jam} the characteristic density, and L the characteristic length. Let $u^* = u/u_{\mathrm{f}}$, $\rho^* = \rho/\rho_{\mathrm{jam}}$, $x^* = x/L$, $t^* = t/(L/u_{\mathrm{f}})$, $a^* = a/u_{\mathrm{f}}$, $Tr^* = Tr/(L/u_{\mathrm{f}})$, $U_{\mathrm{e}}^* = U_{\mathrm{e}}/u_{\mathrm{f}}$, $\Phi_{\mathrm{free}}^* = \Phi_{\mathrm{free}}/(\rho_{\mathrm{jam}}u_{\mathrm{f}}/L)$, $\Phi_{\mathrm{cmpl}}^* = \Phi_{\mathrm{cmpl}}/(\rho_{\mathrm{jam}}u_{\mathrm{f}}/L)$, and $f^* = f/(u_{\mathrm{f}}^2/L)$, which are the dimensionless speed, density, length, time, sonic speed, time delay, equilibrium speed function, free LC source term, compulsive LC source term, and free LC viscosity term. Substituting these into Eq. (7), the dimensionless model equations can be obtained as follows (the symbol $*$ is dropped):

$$\begin{cases} \dfrac{\partial \rho_l}{\partial t} + u_l \dfrac{\partial \rho_l}{\partial x} + \rho_l \dfrac{\partial u_l}{\partial x} = \sum\limits_{l'=l\pm1} \Phi_{\mathrm{free},l'l} - \Phi_{\mathrm{cmpl},l}, \\ \quad \dfrac{\partial u_l}{\partial t} + u_l \dfrac{\partial u_l}{\partial x} + \dfrac{a^2}{\rho_l}\dfrac{\partial \rho_l}{\partial x} \\ = \dfrac{1}{Tr}(U_{\mathrm{e}} - u)_l + f_{\mathrm{viscous},l} + \dfrac{u_l}{\rho_l}\Phi_{\mathrm{cmpl},l}, \\ \Phi_{\mathrm{free},l'l} = C_1^*\left[\rho_{l'}u_{l'}\max(u_l - u_{l'},0) + \rho_l u_l \min(u_l - u_{l'},0)\right] \\ \qquad\qquad + C_2^*\left[\rho_{l'}\max(\rho_{l'} - \rho_l,0) + \rho_l \min(\rho_{l'} - \rho_l,0)\right], \\ f_{\mathrm{viscous},l} = \dfrac{1}{\rho_l}\sum\limits_{l'=l\pm1}\Phi_{\mathrm{free},l'l} \cdot \begin{cases} (1 - u_l), & \rho_l \le 0.2, \\ \left(-\dfrac{1}{4} - u_l\right), & \rho_l > 0.2, \end{cases} \end{cases} \tag{12}$$

where the dimensionless parameters are

$$C_1{}^* = u_{\mathrm{f}}LC_1, \quad C_2{}^* = \frac{\rho_{\mathrm{jam}}L}{u_{\mathrm{f}}}C_2. \tag{13}$$

2.3. Empirical study

The parameter A in Eq. (9), according to its definition, can be estimated by counting the number of vehicles that came through an off-ramp in a certain period of time on a real road section. We choose the Guoding Rd. off-ramp on the Shanghai Middle Ring Expressway, China, as our observation object. This off-ramp is 3.5 km away from the nearest off-ramp upstream at Guangyue Rd. The traffic between the two off-ramps is mostly isolated and thus will not be influenced by the traffic environment outside. It is a four-lane section.

We repeate the experiment three times, each lasting half an hour. The number of vehicles passing from the off-ramp is counted and recorded every 5 min. Results are shown in Table 1. In this table, the maximum value is in boldface and its corresponding compulsive LC intensity is denoted as $A_{\max}$. Let the characteristic length $L = 3$ km, then $A_{\max}$ values of the three entries are respectively as follows: 116 veh/(h $\cdot$ km), 120 veh/(h $\cdot$ km), and 92 veh/(h $\cdot$ km), and for all the entries, $A \in [40,120]$ veh/(h $\cdot$ km). Therefore, the upper limit of compulsive LC intensity on any four-lane highway is given as $A \le 120$ veh/(h $\cdot$ km).

Table 1. Empirical data for the estimation of A.

Set 1 2015.4.6, Monday rainy to cloudy		Set 2 2015.4.10, Friday sunny		Set 3 2015.4.21, Tuesday sunny	
Starting at	No. of vehicles	Starting at	No. of vehicles	Starting at	No. of vehicles
14:13	10	14:13	24	17:42	16
14:18	17	14:18	24	17:47	20
14:23	16	14:23	26	17:52	19
14:28	25	14:28	25	17:57	16
14:33	**29**	14:33	24	18:02	**23**
14:38	21	14:38	**30**	18:07	19

However, traffic conditions may differ from those of this experiment, depending on the local flow rate, the scale of the studied off-ramp, the average demand for leaving from the off-ramp, and how many lanes that section consists of. Therefore, when using our model for simulation, the value of A should be determined accordingly with reference to our empirical data. For example, in the simulation of the traffic on a two-lane road, the value of A should be half that on a four-lane road, that is, $A \in [20,60]$ veh/(h $\cdot$ km).

3. Numerical simulation method

Now we present a numerical simulation method for the model to study the off-ramp problem. The model equations are Eqs. (12), (8), and (9). Hereinafter, a two-lane road section is taken in each numerical case, that is, the number of lanes $n = 2$.

The finite difference scheme is designed by referring to Refs. [21] and [22]. Use the forward difference formula for time derivatives, the backward difference formula for spatial derivatives with the coefficient of speed, and the forward difference formula for spatial derivatives with the coefficient of density, then the scheme will be obtained as follows (subscript l is omitted):

$$\begin{aligned}\rho_i^{n+1} &= \rho_i^n - \frac{\Delta t}{\Delta x}[u_i^n(\rho_i^n - \rho_{i-1}^n) + \rho_i^n(u_{i+1}^n - u_i^n)] \\ &\quad + \Delta t\left(\sum_{l'\neq l}\Phi_{l'l} - \Phi_{\mathrm{compl},l}\right)_i^n, \end{aligned} \tag{14}$$

$$\begin{aligned}u_i^{n+1} &= u_i^n - \frac{\Delta t}{\Delta x}\left[u_i^n(u_i^n - u_{i-1}^n) + \frac{a^2}{\rho_i^n}(\rho_{i+1}^n - \rho_i^n)\right] \\ &\quad + \Delta t\left[\frac{1}{Tr}(U_{\mathrm{e}} - u) + f_{\mathrm{viscous},l} + \frac{u}{\rho}\Phi_{\mathrm{cmpl},l}\right]_i^n. \end{aligned} \tag{15}$$

(Note: here n represents the number of spatial grids, which is different from the meaning of the number of lanes, as defined above.)

When using the discretization scheme for simulation, let $\mathrm{d}x = 0.05$, $\mathrm{d}t = 0.005$, $J = 20$, and $N = 2000$, where J and N are the total number of spatial grids and temporal steps respectively. Constants are set to be $Tr = 0.02$, $a = 0.4$, $x_0^* = 0.6$, $\gamma^* = 15$, and $\beta^* = 150$. According to Subsection 2.1, we set $\alpha_2 = 8.6824$ and $\alpha_1 = 0.8682$ (assuming that only 10% of the vehicles in the first lane wish to exit from this off-ramp). Characteristic values are set to be $L = 3$ km, $\rho_{\mathrm{jam}} = 143$ veh/km, and $u_{\mathrm{f}} = 105$ km/h.

As for the boundary conditions, let all profiles keep unchanged at the inlet, and use the Neumann condition at the outlet, then we will have

$$\rho_0^{n+1} = \rho_0,\ u_0^{n+1} = u_0,\ \rho_J^{n+1} = \rho_{J-1}^{n+1},\ u_J^{n+1} = u_{J-1}^{n+1}. \tag{16}$$

4. Case study

Three numerical cases of the off-ramp problem are studied to demonstrate the reasonability and the applicability in equilibrium and non-equilibrium traffic of our model. Figure 2 illustrates the sketch of a two-lane road section with an off-ramp adopted in these cases.

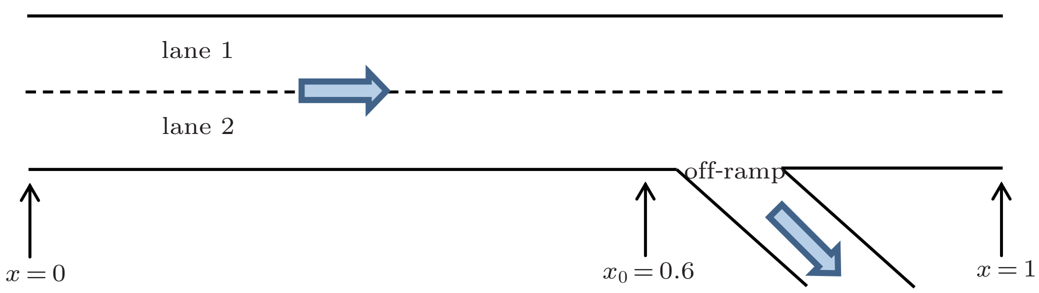

Fig. 2. (color online) Sketch of the off-ramp problem.

4.1. Case 1: Low-density state

Case 1 is designed to validate the model and the discretization scheme, and also to study LC influence on the traffic environment. Suppose that traffic density is low on both lanes, and the speed and density on both lanes are evenly distributed. The initial conditions are

$$\rho_{10} = 0.10,\ \ \rho_{20} = 0.12,\ \ u_{10} = 0.90,\ \ u_{20} = 0.88. \quad (17)$$

Equilibrium speed Ue is given by the dimensionless Greenshields speed-density relationship:[23]

$$Ue(\rho) = 1 - \rho. \quad (18)$$

Calculate the following four conditions:

no LC: $C_1^* = 0$, $C_2^* = 0$, $A = 0$ veh/(h·km);

pure free LC: $C_1^* = 0.25$, $C_2^* = 1.5$, $A = 0$ veh/(h·km);

pure compulsive LC: $C_1^* = 0$, $C_2^* = 0$, $A = 35$ veh/(h·km);

free + compulsive LC: $C_1^* = 0.25$, $C_2^* = 1.5$, $A = 35$ veh/(h·km).

Numerical results of the four conditions are drawn in Fig. 3 to Fig. 6.

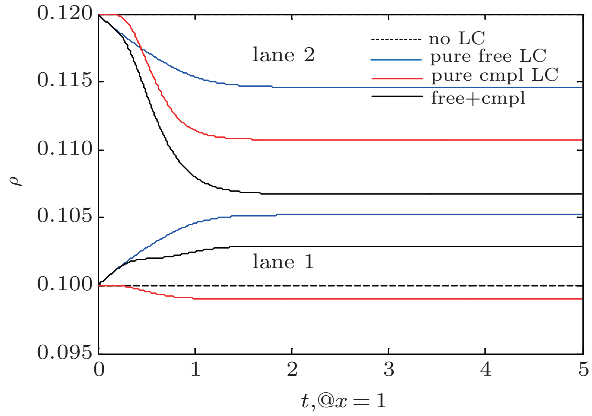

Fig. 3. (color online) ρ–t in low density state.

Figures 3 and 4 show how the traffic state evolves against time at the end of the road section ($x = 1$). The equilibrium state is achieved after $t = 1.7$ (about 2.9 s).

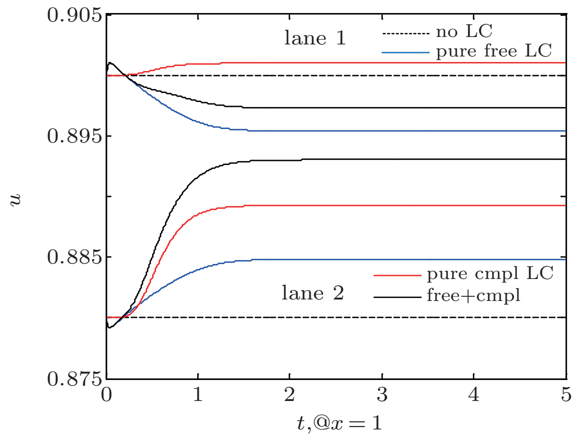

Fig. 4. (color online) u–t in the low density state.

Then figures 5 and 6 show the equilibrium traffic states at $t = 5$. Three parts with distinct patterns can be found in the figures.

(i) From $x = 0$ to $x = 0.3$, the influence of free LC is dominant, which makes speeds and densities of the two lanes converge towards each other, while compulsive LC has little influence on this part.

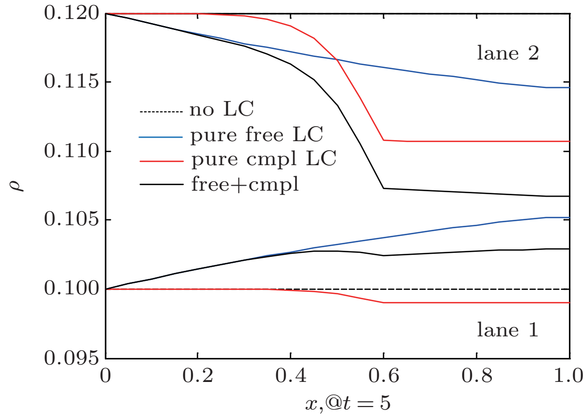

Fig. 5. (color online) ρ–x in low density state.

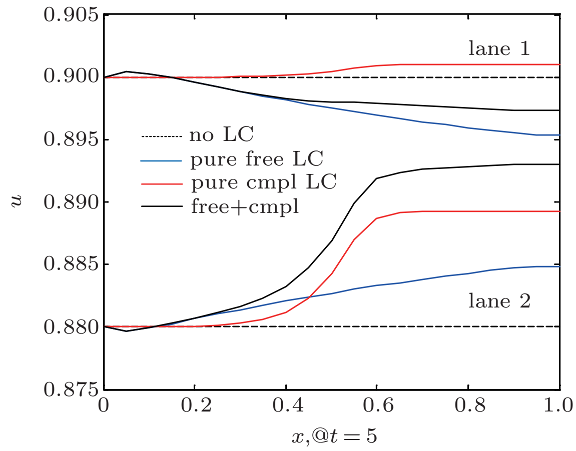

Fig. 6. (color online) u–x in low density state.

(ii) From $x = 0.3$ to $x = 0.6$, compulsive LC starts to affect traffic. As the influence turns stronger downstream, the speed of lane 2 drops dramatically and its speed increases, while the speed and density of lane 1 have similar but slighter changes. The influence of free LC still exists, but becomes less important than that of compulsive LC.

(iii) From $x = 0.6$ to $x = 1$, which is the downstream of the off-ramp, the influence of free LC becomes dominant again, as no compulsive LC happens there. However, the equilibrium state changes due to the influence of the compulsive LC upstream.

It is shown in this case that our model simulates traffic with free and compulsive LC well at low density, and the pure free LC case yields the same results as the low density case in Ref. [19] (at medium and high density, our results of the pure free LC case agree with those in Ref. [19] as well). Two kinds of LCs show different influence patterns in the traffic state. The discretization scheme is stable, and the parameters are well set so as to obtain reasonable results.

4.2. Case 2: Medium- and high-density states

The model (12) should adopt different values of equilibrium speed Ue when simulating medium- and high-density traffic states. In these cases, the dimensionless equilibrium speed Ue is given by Payne's function:

$$Ue = \min\{1, 1.94 - 6\rho + 8\rho^2 - 3.93\rho^3\}. \quad (19)$$

The initial conditions for medium-density state are

$$\rho_{10} = 0.38,\ \rho_{20} = 0.40,\ u_{10} = 0.60,\ \text{and}\ u_{20} = 0.55. \quad (20)$$

We simulate a free-and-compulsive LC situation with the model. Let $A = 30$ veh/(h · km), $C_1^* = 0.25$, and $C_2^* = 1.5$.

Figures 7 and 8 show the equilibrium traffic states at $t = 70$. The speed curves of the two lanes intersect at $x = 0.25$, after which the speed of lane 2 drops lower than that of lane 1. In the downstream off-ramp, speed curves of the two lanes converge. A similar pattern exists in the density curve in Fig. 8, as the density of lane 2 increases over that of lane 1 after $x = 0.25$, and after crossing the off-ramp, the density curves of the two lanes converge.

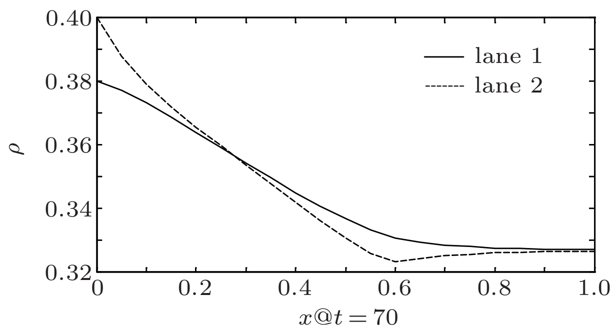

Fig. 7. ρ–x in medium density state.

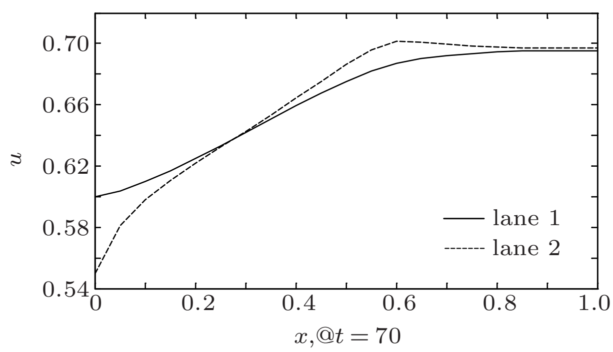

Fig. 8. u–x in medium density state.

The high-density state usually exists in congested urban traffic, where almost every space available is occupied and the speeds and densities of the two lanes are very close. Thus the initial conditions for the high-density state are given as

$$\rho_{10} = 0.70,\ \rho_{20} = 0.70,\ u_{10} = 0.31,\ \text{and}\ u_{20} = 0.31. \quad (21)$$

As traffic becomes congested, the compulsive LC happens less frequently. Let $A = 20$ veh/(h · km).

Figures 9 and 10 show the equilibrium traffic states at $t = 100$. The changing patterns of the curves are similar to those of medium-density traffic, except that the intersection point changes to $x = 0$ due to the initial conditions.

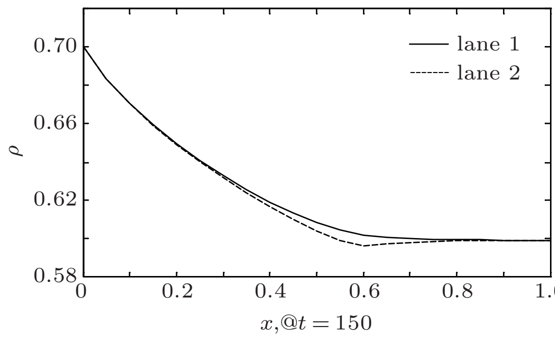

Fig. 9. ρ–x in high density state.

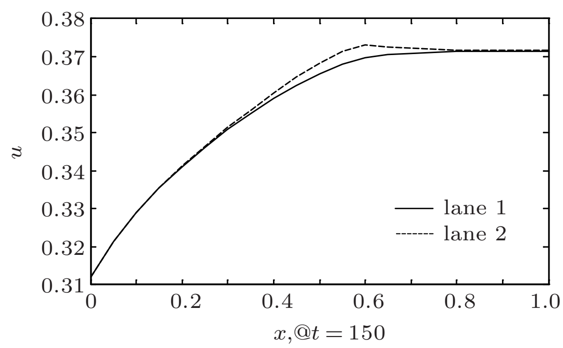

Fig. 10. u–x in high density state.

Figures 7–10 show the different influences of free LC and compulsive LC at the upstream and downstream off-ramp in the medium-/high-density state. In these states, the initial speeds and densities of the two lanes are very close. Therefore, the need for free LC drops down, while the need for compulsive LC remains the same. As vehicles move into the auxiliary lane from lane 2 (compulsive LC behavior), their density will decrease largely and their speeds will increase. This leads to a larger speed/density difference between the two lanes, and then in the downstream off-ramp, the effect of free LC still survives and drives the speed/density of the two lanes close to each other.

4.3. Case 3: Non-equilibrium traffic state

Generally speaking, a second-order differential equation model can simulate the non-equilibrium traffic state well. Therefore, we design Case 3 to study our model in a non-equilibrium state simulation. Take the same settings as the free + compulsive LC condition in Case 1, except that the inlet boundary condition (16) is changed into

$$\rho_0^n = \rho_{\mathrm{eq},0}\left(1+0.15\sin\left(\frac{2\pi}{T_p}n\right)\right), \quad u_0^n = \frac{\rho_{\mathrm{eq},0}}{\rho_0^n}u_{\mathrm{eq},0}, \quad (22)$$

where $\rho_{\mathrm{eq},0}$ and $u_{\mathrm{eq},0}$ are the constant entrance boundary condition values in Case 1, and T_{p} is the dimensionless time period of the sinusoidal function under the assumption of $T_{\mathrm{p}} = 2000$. The simulation results are shown in Figs. 11 and 12. Density and speed curves at $x = 0$, $x = 0.2$, $x = 0.4$, $x = 0.6$, and $x = 1$ are drawn in the same figure for comparison.

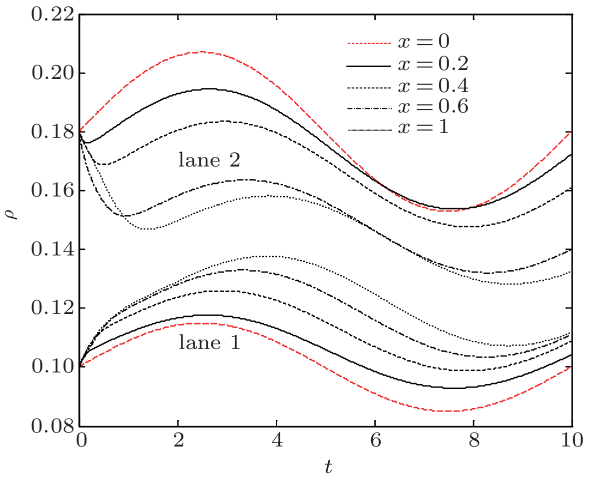

Fig. 11. (color online) ρ–t in non-equilibrium state.

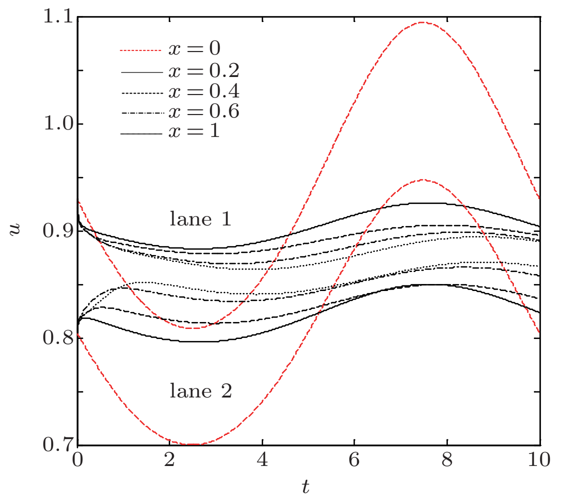

Fig. 12. (color online) u–t in non-equilibrium state.

It can be seen in the figures that as the phase of the fluctuation at the inlet moves downstream, the amplitude reduces. This is caused by the free LC, which is in accordance with the second case from Ref. [19]. On the other hand, at $x = 0.4$, $x = 0.6$, and $x = 1$ respectively, an overall trend that density drops and speed increases can be seen, with the scenario at $x = 0.6$ being the most evident. This is caused by the compulsive LC, which brings down the traffic flow of the road section. This case validates the applicability of our model in the simulation of non-equilibrium traffic near an off-ramp.

5. Conclusions

In this paper we establish a new second-order continuum traffic model with the consideration of both free LC and compulsive LC behaviors. For the off-ramp problem, we give a form of the source term of compulsive LC, and determine its value range of the parameter by the empirical data taken from an off-ramp at Shanghai. A discretization scheme for our model is constructed and applied to three numerical cases, which show that the proposed model and discretization scheme can provide reasonable simulations for various traffic states.

The compulsive LC function model is logically derived through the inspection of real traffic, however, we are interested in looking into how this model can be verified and how the parameters could be determined through empirical data in the future.

In this paper, only one kind of compulsive LC is discussed. However, other kinds of compulsive LC behaviors exist, such as the keep-right-except-to-pass-rule problem: under certain traffic rules, drivers have to change back to the slow lane after overtaking the car in front of it by using the quick lane.[24,25] Such a situation may be simulated by proposing another suitable $\Phi_{\mathrm{ramp},n}$ term in Eq. (8).

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